B. Sc – I(H), Paper – I, Group – A

Relativistic energy :– Mass-energy relation ($E = mc^2$)

Suppose a force $F = \frac{d(mv)}{dt}$ be acting on a particle of mass 'm' so that its kinetic energy increases.
The gain in K.E will be equal to the work done on the particle.
If the force displaces the particle through a distance 'dr' along its line of action, then the infinitesimal gain in the K.E is

$$dE_K = F.dr = \frac{d(mv)}{dt}.dr$$

$$= v\,d(mv) \quad \left[v = \frac{dr}{dt}\right]$$

If the particle starts from rest (v=0) and acquires velocity 'v' under the action of the force, then the gain in the K.E be the particle will

$$E_K = \int dE_K = \int_0^v v\,d(mv)$$

integration by parts, we get

$$E_K = vmv\Big|_0^v - \int_0^v mv\,dv = mv^2 - \int_0^v \frac{m_0 v\,dv}{\sqrt{1-v^2/c^2}}$$

$$= \frac{m_0 v^2}{\sqrt{1-v^2/c^2}} + m_0c^2\sqrt{1-v^2/c^2} - m_0c^2$$

$$= \frac{m_0c^2}{\sqrt{1-v^2/c^2}} - m_0c^2 = mc^2 - m_0c^2$$

Thus, $E_K = (m-m_0)c^2$

$$= \frac{m_0c^2}{\sqrt{1-v^2/c^2}} - m_0c^2 \quad (1) \quad \left[\text{where } m = \frac{m_0}{\sqrt{1-v^2/c^2}}\right]$$

equation (1) shows that relativistic K.E.
It shows that the gain in K.E corresponds to an increase in mass.

The quantity m_0c^2 is due to the rest mass of the particle and is called the rest energy or proper energy E_0 of the particle i.e $E_0 = m_0c^2$.
Thus, the total energy of the particle is

$$E = K.E(E_k) + \text{Rest energy } E_0.$$
$$= (m - m_0)c^2 + m_0c^2$$
$$E = mc^2$$

Thus, $E = \dfrac{m_0 . c^2}{\sqrt{1 - v^2/c^2}} = mc^2$

This energy E is called the relativistic energy (total energy) of a particle, having relativistic mass m. This is also known as Einstein's mass energy relation.